SRI LANKAN PHYSICS OLYMPIAD COMPETITION – 2013

Time Allocated : 02 Hours

Calculators are not allowed to use.

Date of Examination : 15 – 06 – 2013 **Index No. : …………………........**

Time : 9.00 a.m. - 11.00 a.m.

INSTRUCTIONS

- **Answer both questions.**
- **Use the papers provided to do all the derivations.**
- **At the end of each question an answer sheet is provided for you to write down the corresponding final expressions.**
- **At the end of the exam, handover the full question paper together with the final answer sheets.**
- **Handover the papers used to workout the problems separately with your index number written on each paper.**
- **Do not detach any sheet from the question paper.**

(1) AN ILL FATED SATELLITE

Consider a geosynchronous communications satellite of mass *m* placed in an equatorial circular orbit of radius r_0.

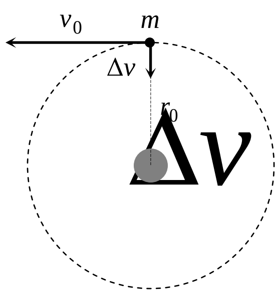
1.1 **Derive an expression for r_0 in terms of *g*, *R* (radius of the earth), and T_0 (length of the day).**

1.2 **Derive an expression for the velocity v_0 of the satellite in terms of *g*, *R*, and r_0.**

1.3 **Obtain an expression for its angular momentum L_0, in terms of *m*, *g*, *R*, and v_0.**

1.4 **Obtain an expression for its total mechanical energy E_0, in terms of v_0, and *m*.**

When the satellite is in this geosynchronous circular orbit, due to an error by the ground controllers causes the engine of the satellite to be fired accidently. The thrust happens to be directed towards the earth and, despite the quick reaction of the ground crew to shut the engine off, an unwanted velocity variation Δv is imparted on the satellite (see Figure F-1). Let us denote this boost by the boost parameter $\beta = \Delta v / v_0$. The duration of the engine burn is always negligible with respect to any other orbital times, so that it can be considered as instantaneous.



F-1

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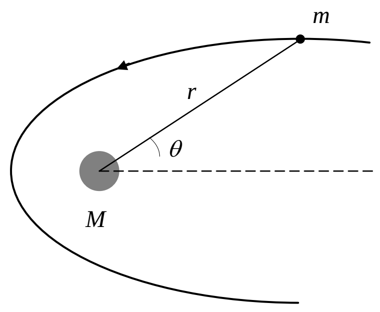
HINT TO WORK OUT THE REST OF THE PROBLEM

Under the action of central forces obeying the inverse-square law, bodies follow trajectories described by ellipses, parabolas or hyperbolas. In the approximation *m* << *M* the gravitating mass *M* is at one of the focuses. Taking the origin at this focus, the general polar equation of these curves can be written as (see Figure F-2)

$$r(\theta) = \frac{l}{1-\varepsilon\ cos\theta}$$



F-2

where *l* is a positive constant named the semi-latus-rectum and ε is the eccentricity of the curve. In terms of constants of motion:

$$l = \frac{L^2}{GMm^2} \qquad \textbf{and} \qquad \varepsilon = \left(1+\frac{2EL^2}{G^2M^2m^3}\right)^{1/2}$$

where *G* is the gravitational constant, *L* is the angular momentum of the orbiting mass, with respect to the origin, and *E* is its total mechanical energy. (i) If $0 \le \varepsilon < 1$, the curve is an ellipse (circle for $\varepsilon = 0$).

(ii) If $\varepsilon = 1$, the curve is a parabola. (iii) If $\varepsilon > 1$, the curve is a hyperbola.

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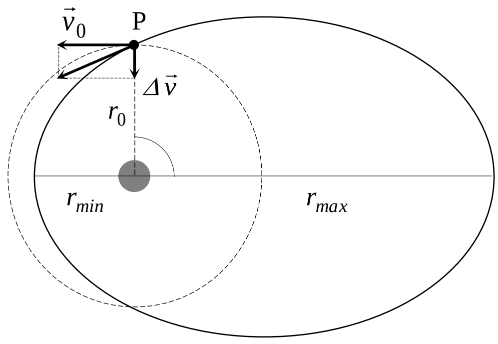
Suppose $\beta < 1$. The new trajectory of the satellite is shown in Figure F-3.



F- 3

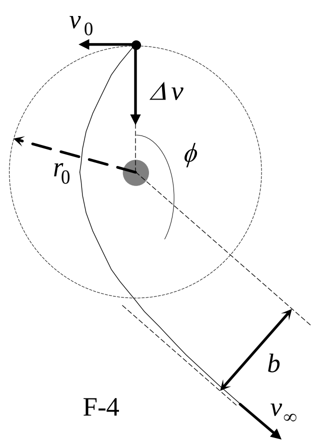
2.1 Obtain an expression for the satellite's new total mechanical energy *E*, in terms of v_0, *m* and *β*.

2.2 Using the expressions given in the hint, determine the parameters of the new orbit, semi-latus-rectum l and eccentricity ε, in terms of r_0 and *β*. (Remember that angular momentum is conserved)

2.3 Give the expressions of the r_{min} (closest distance to the earth) and r_{max} (furthest distance from the earth) in terms of r_0, and *β*.

3.1 Calculate the minimum boost parameter, β_{esc}, needed for the satellite to escape earth gravity.

3.2 Determine in this case the closest approach of the satellite to the earth centre in the new trajectory, r'_{min}, in terms of r_0.



F-4

Suppose $\beta > \beta_{esc}$.

4.1 Determine the velocity of the satellite at the infinity, v_∞, in terms of v_0 and *β*.

4.2 Obtain an expression the "impact parameter" *b* of the escape direction of the satellite in terms of r_0 and *β*. (See Figure F-4).

ANSWER SHEET

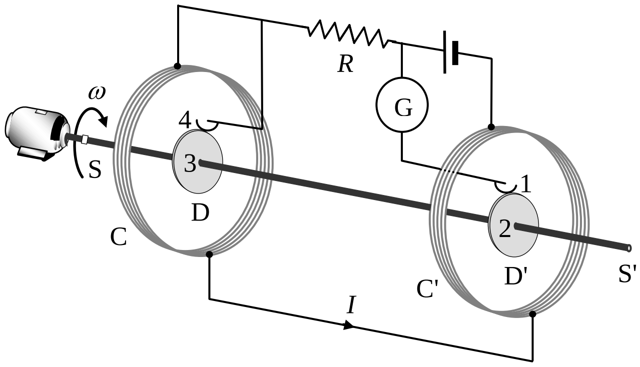
Question	Results	Marks
1.1	$r_0 = \left(\frac{gR^2T_0^2}{4\pi^2}\right)^{1/3}$	03
1.2	$v_0 = R\sqrt{\frac{g}{r_0}}$	03
1.3	$L_0 = \frac{mgR^2}{v_0}$	04
1.4	$E_0 = -\frac{1}{2}mv_0^2$	04
2.1	$E = \frac{1}{2}mv_0^2(\beta^2 - 1)$	04
2.2	$l = r_0$ $\epsilon = \beta$	04 04
2.3	$r_{max} = \frac{r_0}{1-\beta}$ $r_{min} = \frac{r_0}{1+\beta}$	04 04
3.1	$\beta_{esc} = 1$	04
3.2	$r'_{min} = \frac{r_0}{2}$	04
4.1	$v_\infty = v_0(\beta^2 - 1)^{1/2}$	04
4.2	$b = r_0(\beta^2 - 1)^{-1/2}$	04

(2) ABSOLUTE MEASUREMENTS OF ELECTRICAL QUANTITIES

The technological and scientific transformations underwent during the 19th century produced a compelling need of universally accepted standards for the electrical quantities. It was thought that units of electrical quantities should only rely on the standards of length, mass and time. An intensive experimental work to settle the values of these units was developed from 1861 until 1912.

Determination of the ohm (Rayleigh, Sidgwick).

The experimental setup is shown in Figure F-1. It consists of two identical metal disks D and D' of radius *b* mounted on the conducting shaft SS'. A motor rotates the set at an angular velocity ω, which can be adjusted for measuring *R*. Two identical coils C and C' (of radius *a*, and with *N* turns each) surround the disks. They are connected in such a form that the current *I* flows through them in opposite directions. The whole apparatus serves to measure the resistance *R*.



F-1

1.1 Assume that the current *I* flowing through the coils C and C' creates a uniform magnetic field of flux density *B* around D and D', equal to the one at the centre of the coil. Write down an expression for *B* in terms of *I*, *N*, and *a*.

1.2 Derive an expression for the e.m.f., ε', induced on each disk (between the center and the rim) by the magnetic filed in terms of *B*, *b*, and ω.

1.3 Write down an expression for the electromotive force ε induced between the rims 1 and 4, in terms of *N*, *b*, *I*, *a*, and ω, assuming that the distance between the coils is much larger than the radius of the coils and that $a >> b$.

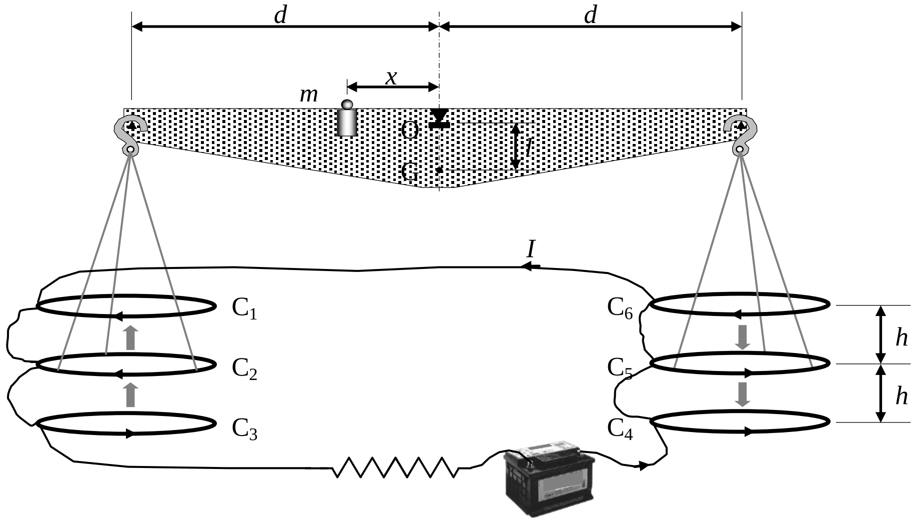
The disks are connected to the circuit by brush contacts at their rims 1 and 4. The galvanometer G detects the flow of the current through the circuit 1-2-3-4.

1.4 The resistance *R* is measured when G reads zero. Give *R* in terms of the physical parameters (*N*, *b*, *a*, and ω) of the system.

Determination of the ampere

Passing a current through two conductors and measuring the force between them provides an absolute determination of the current itself. The "Current Balance" designed by Lord Kelvin in 1882 exploits this method. It consists of six identical single turn coils $C_1 \ldots C_6$ of radius *a*, connected in series. As shown in Figure F-2, the fixed coils C_1, C_3, C_4, and C_6 are on two horizontal planes separated by a small distance 2*h*. The coils C_2 and C_5 are carried on balance arms of length *d*, and they are, in equilibrium, equidistant from both planes.

The current *I* flows through the various coils in such a direction that the magnetic force on C_2 is upwards while that on C_5 is downwards. A mass *m* at a distance *x* from the fulcrum O is required to restore the balance to the equilibrium position described above when the current flows through the circuit.



F-2

Assume that C_1, C_3, C_4, and C_6 are long straight wires carrying parallel currents. Further assume that C_2 and C_5 are straight wires of length $2\pi a$.

2.1 Write down an expression for the force F on C_2 due to the magnetic interaction with C_1 in terms of a, I, and h.

2.2 The current I is measured when the balance is in equilibrium. Derive an expression for I in terms of the physical parameters (m, h, x, a, and d) of the system. The dimensions of the apparatus are such that we can neglect the mutual effects of the coils on the left and on the right.

ANSWER SHEET

Question	Results	Marks
1.1	$B = \mu_0 \frac{NI}{2a}$	**05**
1.2	$\epsilon' = \frac{1}{2} B\omega b^2$	**10**
1.3	$\epsilon = \mu_0 \frac{NI\omega b^2}{2a}$	**10**
1.4	$R = \mu_0 \frac{N\omega b^2}{2a}$	**05**
2.1	$\boldsymbol{F} = \mu_0 \frac{aI^2}{h}$	**10**
2.2	$I = \left(\frac{mghx}{4\mu_0 ad}\right)^{1/2}$	**10**